# Preface

Mathematics possesses a central importance in our society. It shapes and influences many areas of our daily life, from education and culture via technology and industry to physics and information science, and more. Most obvious and most closest examples to our experiences comprise the design and the evaluation of insurance products and financing schemes, mobile wireless communication, electronic devices for a global positioning system, and much more. In many of these examples, the contribution of mathematics to the development of the functionality was decisive, and there are billions of users, but hardly anybody notices the mathematics involved anywhere in the final product. Indeed, this phenomenon seems to be even characteristic for mathematics: its rôle in the development of a new technology or device is vital, but cannot be seen in the product itself. This is even more distinctive in intangible concepts and procedures that have been shaped by mathematics, like encryption procedures that make possible online banking, e-mail and the internet, or finance products that are adapted to special market situations. Mathematics is important for the development of technology and industry for many centuries and will continue so for ever, and real-world problems inspire and accelerate mathematical research in many ways.

But there are also further, immaterial, impacts of mathematics on the society. For example, some exciting developments in mathematics raised interesting philosophic questions that are at times discussed in public, like the question about the validity of proofs: how well must a proof be checked, such that one can agree that the assertion has been proved? This is even more difficult to decide for computer-aided proofs, the existence of which has gained quite some popularity, at least within the mathematical community (think of the famous four-colour problem, which says that four colours suffice for colouring a map without giving the same colour to any two neighbouring areas). Popularity even outside the community is enjoyed by questions about the quest for solutions to famous open problems like the Clay Millennium Problems (www.claymath.org/millennium-problems). Another much-discussed aspect is the general rôle of mathematics in education, as it inspires many people, including pupils, as a freetime-occupation and stimulates them to further investigation of their own.

When we talk about mathematics in society, then we mean mathematics that has an influence on the daily life of a substantial part of the people, or mathematics that is an integral part of our society in one or another way. Furthermore, we mean such kind of mathematics that can be done only by a professional mathematical expert. We talk about the search of new concepts, methods and proofs on a level that necessitates investigation by mathematical researchers (jointly with the efforts of specialists of the application field, if necessary).

As we said at the beginning, much mathematical research whose results are decisive for the wealth of the society and for solutions to its problems is so well hidden that the public

does not know about it, often not even the scientifically interested layman. Likewise, several application fields are generally known, but not the way in which mathematics makes an important contribution. There are many reasons for this unfortunate situation. One is of course that an understanding of the problem requires some least level of preparatory training in the application field, which the layman typically does not have. Another one, and may be the decisive one, is that the mathematics involved is quite abstract and specialised and needs even more training in mathematics. And a third main reason is the lack of expert people who are willing to take time for explaining things in a way that the scientifically interested layman can understand.

This volume is an attempt to rectify the situation. It tells success stories about some important examples of mathematics in society: a real-world problem to be solved, mathematical difficulties that the problem poses, and the rôle that mathematics plays in the solution. All these stories are written by eminent and experienced mathematicians, who shaped, and continue to shape, the application field that they describe. The goal was to cover a great diversity of application areas, and to explain things on a non-technical level, such that the scientifically interested layman can take home quite some message. Certainly also professionals, both those in mathematics and those in the application area, will greatly benefit. But, given that fifteen (rather diverse!) subjects are highlighted in this collection, nobody can be an expert in all of them, but every scientifically interested will find something that she or he is able to follow and to enjoy.

It is always difficult to talk about mathematics and its applications to real-world problems; even more, if readers without specialised education are addressed. The matter is generally difficult, the author's and the reader's background differ a lot, and also the readership is rather diverse: from expert mathematicians to mathematicians that work in other areas, and from experts on the application field to the layman who is just interested in science. All the authors of this volume have tried their best to address at least more than one of these groups, and some of them even aimed at the latter type of reader, which is definitely the most difficult task.

Being passionate mathematicians, the authors of this collection certainly present also a great deal of mathematical material; how else can you demonstrate its necessity and its benefits better? It may be the a exciting aspect of each of the essays that the authors' personal tastes shine through the choices of the subjects, through the ways of presentation and through the comments. It is always interesting to learn about the personal view of the expert at the things, even more as the editorial board encouraged the contributors to reveal some of that, and many of the authors gratefully used the opportunity to do so.

What are these application fields that we decided to present here? It was our purpose to present a most diverse collection of aspects and areas of mathematics that have shaped and continue to shape our society. Here is a survey.

Let us begin with a fundamental question about the interaction between mathematics and the society: the question about how a mathematical expert should write about his field for the public, such that (s)he will be understood and reaches her/his readership – precisely the situation in which all the authors of this book are! Read the advices and entertaining examples of how to do that and how one should not do that by George Szpiro! – We proceed with more abstract approaches to the question how mathematics enters and influences society. The enormous computer power that mankind is now able to use produced new ways of production of new mathematics, namely by computer-assisted proofs and by automated verification of

existing, complex proofs and falsification of conjectures. Donald Bailey and Jonathan Borwein present deep and recent thoughts about this new field called *experimental mathematics*, and touch also some fundamental questions like the one about when a proof can be considered valid.

One of the reasons that mathematics is esteemed so high in our society is presumably its rôle in the education as a science that is most amenable to teaching methodologies that appeal to the natural wish of humans to find out by own doing. The conscience for this in our society is significantly increasing only for a few years, but it has lead to (and has been increased by) the foundation of science centers, hands-on museums, or even special mathematics museums. One of the most well-known educational centers of this kind (at least in Germany), the *Mathematikum*, was founded some ten years ago in Gießen by our author Albrecht Beutelspacher. In his contribution, he describes its concept and the reasons for its success. Furthermore, he gives an account on the highly interesting question, whether or not the institution of all these mathematics museums was worthwhile and how they changed the attitude of the population towards mathematics.

Let us proceed with application areas that have a notable random component. In finance, some few years ago, a big global crisis moved the world, and still the unspoken question remains in the room, whether or not the crisis happened *because* of the work of mathematicians or *in spite of* it. Read Walter Schachermayer's opinion about that! He also reports on the interesting history of the first applications of the Brownian motion to finance some hundred years ago. – One of the ubiquitous parts of applied mathematics is the field of statistics, whose goal is the probabilistic description of real-world phenomena. In the research of cancer, nowadays many approaches make use of an enormously huge amount of complex data. A solution to the problem how to extract useful information from these big data lies by no means in a strategy of brute-force computing with stronger and stronger computer power, but in the development of more and more sophisticated statistical methods, notably handling the high-dimensionality of the structure in the data. Aad van der Vaart and Wessel van Wieringen give some insight in the mathematical aspects of these methods. – Another application field of statistics is filtering theory within engineering. The story of this method and its historical successes is told by Ofer Zeitouni. – Probability plays also an important rôle in population models with application in biology. Especially in recent years, new models have been introduced that take care of the latest state of understanding of the evolution of biological populations. Jean Bertoin presents a pedagogical summary of some of the models that mathematical biologists work on most actively at the moment.

One of the areas with the oldest, most immediate, and most discussed connections with mathematics is physics. Already the ancient Greeks deeply thought about the laws that underly all the matter and the mathematical way to describe them. By example of the Second Law of Thermodynamics and the concept of entropy, Jürg Fröhlich brings to the reader the spirit of the old quest for the understanding of fundamental laws that control a great deal of actions around us.

Another large and classical part of mathematics, even the oldest, but still very modern and, one can say, ubiquitous field is geometry, which is more up-to-date than one might think. Actually, in architecture it plays nowadays an equally active rôle as thousands of years ago. More specifically, Hellmut Pottmann and Johannes Wallner concentrate on problems from discrete differential geometry that an architect has to solve if (s)he plans to design a piece of freeform architecture. Many pictorial examples from the practice illustrate the interplay between the

mathematical theory and the intended form of the building. – While this article is about the geometry of objects that humans want to shape on their own, Christiane Rousseau concentrates on those shapes that appear in natural morphologies, for example animals and plants, and follow rules of geometry that lead to beautiful and functional solutions. She brings the rules underlying the natural shaping forces to the surface and explains why similar geometric forms appear in most diverse connections in the nature. Striking relations with fascinating mathematical objects like the Koch snowflake appear in a new context.

Let us come to applications of mathematics in industry. First of all, does "industrial mathematics" exist at all? One of the most experienced experts in this field, Helmut Neunzert, raises this question and extends it to the question "industrial mathematics versus academic mathematics". This is only a starting point for a large-scale survey on the history and the current situation in the relationship of mathematics, as is carried out in applied research, and industrial mathematics that is meant to solve explicit tasks, including a lot of philosophical considerations and personal statements! – Telecommunication is a concrete industrial field in which mathematics has much to do and to say, and its contributions are enormously diverse and versatile. Holger Boche and Ezra Tampubolon concentrate on a particular question that is ubiquitous in the theory and praxis of data transmission and can be resolved only by an ingenious use of highly developed mathematics: how can one handle the huge differences in the amplitude of the transmission of a signal by means of an orthogonal transmission scheme? They demonstrate that this annoying problem can be settled by use of some parts of mathematics that are considered to be quite pure, like additive combinatorics, but also more applied disciplines like functional and harmonic analysis.

Information security poses severe challenges for the society of the future. Cryptography now provides ramified techniques to deal with these challenges. It constitutes a wide field which is now heavily based on methods from a variety of mathematical disciplines, notably complexity theory and number theory. Claus Diem explores these connections. Also, the limitations of the mathematical approach to real life security are critically addressed.

A field that would be not guessed as a field of mathematical application is the field of politics, more precisely, voting systems. Actually, there are hardly any two votes that are carried out under precisely the same set of rules, and slight changes in the voting rules can lead to surprising changes in the result, and not only theoretically. Werner Kirsch gives a flavour of a mathematical concept of voting systems, its benefits, the effects that it contains and the conclusions that one can draw within the concept. The consequences of some theoretical results for the society can be pretty immediate, as he illustrates by means of historical examples.

The reader might have noticed that a field that is generally thought to have high affinity to mathematics has not yet (or only once, see the above mentioned contribution to statistics) been addressed: handling big data. Last, but not least, there is also one essay devoted to this important subject, in the connection of investigation of the climate and the weather. Here it is not possible to make experiments, one has to rely on mathematical descriptions and predictions of the reality. Obviously, huge amounts of data are available, but the biggest problems come from the enormous span of scales of the meaning of the data. Jörn Behrens describes the established mathematical models and methods in geoscience; in particular the rôle of the important field of uncertainty: what can mathematics do if we not even know the probability distribution of the unknown quantities in our equation?

This ends our small survey of the articles contained in this volume. Let me express my sincere thanks to the inspiring support of my colleagues that formed a awesome editorial

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# Contents