Wangerin

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Explicationem vulgatam Newtonis annulorum, qui dicuntur, quamquam a summis physicis, Th. Young, Poisson, Fresnel, positam atque in principio sine dubio rectam, non satis diligentem esse existimo. Duae quidem res huic explicationi repugnant. Primo enim secundum interpretationem vulgatam oriuntur annuli ex parallelis lucis radiis interferentibus. Quomodo si res sese haberet, fieri deberet, ut annuli in qualibet a lamina distantia telescopio cernerentur. Nunc vero experimentum docet, non ita rem sese habere, sed annulos cerni non posse, nisi, parvo spatio interjecto, per microscopium sive nudum oculum, qui microscopii munere fungitur. Deinde vero etiam hac de causa apparet, explicationem eam, quae adhuc proposita est, non satis esse accuratam: nisi enim, sicuti adhuc factum est, lamellae superficies inferior plana et superiori parallela statuitur, nisi igitur sphaerae loco superficies scalarum similis proponitur, sed si curvavaturae sphaerae radios reflectentis ratio habetur: jamjam levissima comtemplatio constructionis geometricae auxilio demonstrat, nosque postea calculo demonstrabimus, generaliter fieri non posse, ut a quoque laminae planae puncto duo radii exeant, qui per totam longitudinem concidant atque interferant, sed a quoque puncto exire duos radios divergentes [ubi interim unam in sphaera reflexionem statuimus]. Quos radios postea ostendemus parvum sane angulum continere; sed apparet ex iis, quae diximus, etiamsi ejus rei, quam primo loco opposuimus, rationem non habemus eam, quae adhuc proposita est, interpretationem primam modo approximationem esse putandam, atque talem, in qua de membrorum neglectorum magnitudine nihil omnino docetur. Omnem vero approximationem, in qua membrorum neglectorum valor numerari non potest, scientiae non satisfacere constat.

Quae cum ita sint, proposui mihi, ex novo quodam principio phaenomena annulorum Newtonis explicare. Quod quidem quidem quantum mihi successerit, ex hac dissertatione apparebit. Simul vero pietatis esse existimo, praeceptori illustrissimo F. Neumann hoc loco gratias agere, qui cum me commovit, ut huic rei operam darem, tum ad instituenda experimenta apparatus necessarios mihi concessit.

1. Principium explicationis.

Phaenomenon propositum explicabo ex eodem principio, ex quo explicantur ea diffractionis phaenomena, quae Frauenhoferi sive microscopica dicuntur. Incidat enim (Fig. 1) in planitiem, quae lamellam a superiore parte confinit, fascis radiorum parallelorum, quorum unus AB planitiem in B secat atque parte quadam in lamellam frangitur. Radius fractus BC sphaeram, quae lamellam ab inferiore parte finit, contingit in C atque reflexus in sphaera, in D in medium superius refringitur versus DE. Punctum D vero simul e fasci radiorum incidentium radius FD contingit et versus DG reflectitur. Atque facile apparet, radios DE et DG angulum quendam continere; exeunt igitur a D duo radii divergentes, qui generaliter simul diversa phasi utuntur. Qui radii si lenticula KK excipiuntur, radii fracti in puncto D_1 post lenticulam convenient. Atque si duo radii pari phasi e D exirent, etiam pari phasi in D venirent. Nunc vero duo radii in D propter differentiam viarum non sunt pari in phasi, eademque phasium differentia, quacum in D sunt, etiam in D_1 conveniunt. Punctum D_1 igitur eodem modo oscillat, quo D. Si autem in puncto quodam D duo radii decussantur, secundum principium coexistentiae parvorum motuum alter alterum secat, neutro motu perturbato. Solum quidem decussis punctum duorum impulsium causa, qui in eum exercentur, proprium motum accipiet atque generaliter in ellipsi movebitur. Qui motus in eo tantum casu linearis erit, si phasium differentia multiplum dimidiae undulationis est. Atque secundum ea, quae antea diximus, punctum D_1 eosdem motus aceipiet quos D, quia phasium differentia in D et D_1 eadem est; oritur igitur in D_1 puncti D imago.

Atque si phaenomenon nudo oculo perspicimus, *KK* oculi lenticula est; phaenomenon vero tum modo cerni potest, si punctum $D_{,}$ in retina positum est; itaque necesse est, decussis punctum radiorum retro prorogatorum in amplitudine visus perspicui [quod nos dicimus deutliche Sehweite] sitam esse.

Atque hac explicatione duae dubitationes prioris theoriae expediuntur. Primum enim curvaturae sphaerae ratio habetur; tum vero apparet, annulos non in quolibet intervallo a lamina cerni posse, sed tum modo, si $D_{,}$ in retina positum est, id est, si planities lamellam finiens in amplitudine visus perspicui posita est; — quod quidem cum experientia congruit.

Radii vero *DC* pars modo in *D* frangitur, pars iterum in lamellam reflectitur; quae in sphaera in $C_{,}$ reflexa partim in *H* exit e lamella; ita ut a quoque laminae puncto non duo, sed infinitus numerus radiorum divergentium exeat, quorum intensitas ob iteratas reflexiones paullatim minuitur; itaque horum radiorum in *D* decussantium infinitus numerus efficit, ut *D* itaque etiam $D_{,}$ in ellipsi quadam moveantur, quae ab ea, quam antea diximus, discrepat.

Accedit vero, ut is casus, quem adhuc consideravi, idealis est; in quo ponitur, oculum ipsum esse in eo medio, quo lamella a superiore parte finitur. Revera autem medium superius (Fig. 2) tabula est, duobus planis parallelis finita; quare denique refractio radiorum efficitur; quae quem habeat effectum, postea explicabimus.

2. Radius in sphaera reflexus constituitur coordinatis ejus puncti, in quo e lamella exit.

Lamella finiatur altera parte a plano altera a sphaera. Sphaera planitiem non tangat, sed intersit inter eas intervallum Δ. Planum coordinatarum $x\ y$ concidat cum planitie superiore. Sit R radius sphaerae, aequatio sphaerae est:

$$1)\qquad x^2 + y^2 + (R + \Delta - z)^2 - R^2 = 0$$

Systema radiorum parallelorum incidentium sit:

$$2)\qquad \begin{aligned} az &= c\,(x - x_{,}) \\ bz &= c\,(y - y_{,}) \\ a^2 + b^2 + c^2 &= 1, \end{aligned}$$

ut a, b, c determinantia radiorum sint.

Axium x et y situs in plano superiore arbitrarius est; quem situm ita statuam, ut axis x parallela projectioni radiorum in planum superius. Ex quo sequitur; si φ angulus incidentiae

$$3)\qquad a = \sin\varphi \quad b = O \quad c = \cos\varphi$$

Calculi autem propter symmetriam has valores quantitatum a, b, c ad extremum calculum introducam.

Atque systema radiorum 2) in superiore plano in lamellam ita frangitur, ut omnes radii paralleli maneant, unde existit sytema.

$$4)\qquad \begin{aligned} a_{,}z &= c_{,}\,(x - x_{,}) \\ b_{,}z &= c_{,}\,(y - y_{,}) \end{aligned}$$

$$a = Na_{,},\ b = Nb_{,},\ c_{,} = \frac{1}{N}\sqrt{N^2 - 1 + c^2}$$

ubi N exponens refractionis a superiore medio in inferius est. Nunc vero aliquis systematis (1) radius secat sphaeram in puncto $x_{,,}$, $y_{,,}$, $z_{,,}$ ubi

$$5)\qquad z_{,,} = R + \varDelta - \sqrt{R^2 - (x_{,,}^2 + y_{,,}^2)}$$

Atque si radii hoc loco reflectuntur, oritur systema radiorum reflexorum:

$$6)\qquad \begin{aligned} m\,(z - z_{,,}) &= o\,(x - x_{,,}) \\ n\,(z - z_{,,}) &= o\,(y - y_{,,}) \end{aligned}$$

Coëfficientes m, n, o his conditionibus statuuntur:

1) radius reflexus cum normali sphaerae in puncto $x_{,,}$, $y_{,,}$, $z_{,,}$ eundem angulum continere debet, quem radius incidens. Est igitur

$$\frac{a_{,}.\,x_{,,} + b_{,}.\,y_{,,} + c_{,}\,(z_{,,} - R - \varDelta)}{\sqrt{a_{,}^2 + b_{,}^2 + c_{,}^2}} = \frac{m x_{,,} + n y_{,,} + o\,(z_{,,} - R - \varDelta)}{\sqrt{m^2 + n^2 + o^2}};$$

2) radius reflexus in ea planitie situs est, quae radium incidentem et normalem sphaerae continet; est igitur

$$m\left\{c_{,}\,y_{,,} - b_{,}\,(z_{,,} - R - \varDelta)\right\} + n\left\{a_{,}\,(z_{,,} - R - \varDelta) - c_{,}\,x_{,,}\right\} + o\,(b_1 x_{11} - a_1 y_{11}) = O.$$

Sequitur ex his aequationibus

$$7)\qquad \left\{\begin{aligned} \frac{m}{o} = \frac{1}{N}\Big\{ & x_{,,}\,(z_{,,} - R - \varDelta)\,(a_{,}^2 + b_{,}^2 + c_{,}^2) \\ & + [c_{,}\,y_{,,} - b_{,}\,(z_{,,} - R - \varDelta)]\,[b_{,}\,x_{,,} - a_{,}\,y_{,,}] \\ & \pm [a_{,}\,(z_{,,} - R - \varDelta) - c_{,}\,x_{,,}]\,[a_{,}\,x_{,,} + b_{,}\,y_{,,} + c_{,}\,(z_{,,} - R - \varDelta)]\Big\} \end{aligned}\right.$$

$$
7)\quad \begin{cases}
\frac{n}{o} = \frac{1}{N} \Big\{ y_{,,} (z_{,,} - R - \Delta)(a_{,}^2 + b_{,}^2 + c_{,}^2) \\
\qquad + [a_{,}(z_{,,} - R - \Delta) - c_{,} x_{,,}]\,[b_{,} x_{,,} - a_{,} y_{,,}] \\
\qquad \pm [b_{,}(z_{,,} - R - \Delta) - c_{,} y_{,,}]\,[a_{,} x_{,,} + b_{,} y_{,,} + c_{,}(z_{,,} - R - \Delta)] \Big\} \\
\text{ubi} \\
N = (z_{,,} - R - \Delta)^2 (a_{,}^2 + b_{,}^2 + c_{,}^2) - [c_{,} y_{,,} - b_{,}(z_{,,} - R - \Delta)]^2 \\
\qquad - [a_{,}(z_{,,} - R - \Delta) - c_{,} x_{,,}]^2
\end{cases}
$$

Atque signum superius efficit:

$$
8)\quad \frac{m}{o} = \frac{a_{,}}{c_{,}}, \quad \frac{n}{o} = \frac{b_{,}}{c_{,}},
$$

quod ipsum radium incidentem significat. E signo vero inferiore, si adjicimus conditiones

$$
9)\quad a_{,}^2 + b_{,}^2 + c_{,}^2 = 1, \quad m^2 + n^2 + o^2 = 1
$$

patet:

$$
10)\quad \begin{cases}
m = -a_{,} + 2\frac{x_{,,}}{R}\left[a_{,}\frac{x_{,,}}{R} + b_{,}\frac{y_{,,}}{R} + c_{,}\frac{z_{,,} - R - \Delta}{R}\right] \\
n = -b_{,} + 2\frac{y_{,,}}{R}\left[a_{,}\frac{x_{,,}}{R} + b_{,}\frac{y_{,,}}{R} + c_{,}\frac{z_{,,} - R - \Delta}{R}\right] \\
o = -c_{,} + 2\frac{z_{,,} - R - \Delta}{R}\left[a_{,}\frac{x_{,,}}{R} + b_{,}\frac{y_{,,}}{R} + c_{,}\frac{z_{,,} - R - \Delta}{R}\right]
\end{cases}
$$

Jam si unus e radiis 6) planum superius in puncto $x_{,,,}$, $y_{,,,}$, $z_{,,,} = o$ secat, sequitur

$$
11)\quad \begin{cases}
x_{,,,} = x_{,,} - \frac{m}{o} z_{,,} \\
y_{,,,} = y_{,,} - \frac{n}{o} z_{,,}
\end{cases}
$$

qui radius cum axi z angulum continet, cujus cosinus $= o$.

Jamjam quantitates $m, n, o, x_{,,,}, y_{,,,}$ expressae sunt per $x_{,,}$ et $y_{,,}$; nunc contrario $m, n, o, x_{,,}, y_{,,}, z_{,,}$ explicabo functiones quantitatum $x_{,,,}$ et $y_{,,,}$. Simul quaeritur, an in puncto $x_{,,,}, y_{,,,}$ unus radius e lamella exeat an plures radiorum semel in sphaera reflexorum? Facile intellegitur, tot radios esse, quot radices aequationum 11), si quantitates $x_{,,,}, y_{,,,}$ constantes, $x_{,,}, y_{,,}, z_{,,}$ ignotae putantur, ubi inter $x_{,,}, y_{,,}, z_{,,}$ etiam aequatio 1) valet. Atque primum apparet, innumerabilem radiorum numerum non esse, nisi aequatio 1) sequitur e duabus aequationibus 11), quod nunquam fieri facile intelligitur. Si vero e tribus aequationibus 1) et 11) $y_{,,}$ et $z_{,,}$ eliminantur, restat pro $x_{,,}$ aequatio ordinis decimi sexti; sedecim igitur radii in puncto dato $x_{,,,}, y_{,,,}$ convenire videntur; quorum radiorum dimidium ab initio omittendum, quia aequationis quadraticae (1) una radix respicienda. Reliquas autem radices in unam reduci, hoc modo sequitur. Si enim (pro $\varDelta = o$) a puncto $x_{,,,}$, $y_{,,,}$ conus sphaeram tangens ponitur, radiorum semel reflexorum ii modo in puncto $x_{,,,}$, $y_{,,,}$ convenire possunt, quorum punctum reflexionis est intra eum circulum, in quo sphaera a cono tangitur. Quare eae modo aequationis superioris radices respiciendae, quae ad puncta pertinent intra circulum tangentem sita; id est, cum $\varDelta$ semper quantitas secundi ordinis ad $x_{,,,}$, $y_{,,,}$ sit, esse debet

$$\sqrt{x_{,,}^2 + y_{,,}^2} < 2 \sqrt{x_{,,,}^2 + y_{,,,}^2}.$$

Jam vero, si in in aequationibus (11) pro z valorem 5) introducimus et secundum potestates quantitatum $x_{,,}$, $y_{,,}$ evolvimus efficitur:

$$12)\left\{\begin{aligned}\frac{x_{,,,}}{R} &= \frac{x_{,,}}{R} + \frac{a_,}{c_,}\frac{\varDelta}{R} + \frac{1}{2}\frac{a_,}{c_,}\frac{x_{,,}^2 + y_{,,}^2}{R^2} \\ &\quad + \left[\frac{x_{,,}}{R}\left(1 + \frac{a_,^2}{c_,^2}\right) + \frac{y_{,,}}{R}\frac{a_, b_,}{c_,^2}\right] \cdot \left[\frac{\varDelta}{R} + \frac{x_{,,}^2 + y_{,,}^2}{R^2}\right] \\ \frac{y_{,,,}}{R} &= \frac{y_{,,}}{R} + \frac{b_,}{c_,}\frac{\varDelta}{R} + \frac{1}{2}\frac{b_,}{c_,}\frac{x_{,,}^2 + y_{,,}^2}{R^2} \\ &\quad + \left[\frac{x_{,,}}{R}\frac{a_, b_,}{c_,^2} + \frac{y_{,,}}{R}\left(1 + \frac{b_,^2}{c_,^2}\right)\right] \cdot \left[\frac{\varDelta}{R} + \frac{x_{,,}^2 + y_{,,}^2}{R^2}\right]\end{aligned}\right.$$

Atque si has series convertas, tertiis jam potestatibus neglectis, efficiatur:

$$\frac{x_{,,}}{R} = \xi - \frac{a_,}{c_,}\frac{\varDelta}{R} - \frac{1}{2}\frac{a_,}{c_,}\chi$$

$$\frac{y_{,,}}{R} = \eta - \frac{b_,}{c_,}\frac{\varDelta}{R} - \frac{1}{2}\frac{b_,}{c_,}\chi$$

$$\chi = \frac{2c_,}{a_,^2 + b_,^2}\left\{c_, + a_,\xi + b_,\eta\right\}$$
$$\pm \frac{2c_,}{a_,^2 + b_,^2}\sqrt{(c_, + a_,\xi + b_,\eta)^2 - (a_,^2 + b_,^2)(\xi^2 + \eta^2)}$$

ubi $\xi = \frac{x_{,,,}}{R}$, $\eta = \frac{y_{,,,}}{R}$.

Signum vero (+) pro χ talem valorem efficeret, pro quo $\sqrt{\left(\frac{x_{,,}}{R}\right)^2 + \left(\frac{y_{,,}}{R}\right)^2}$ non amplius eodem ordine esset, quo $\sqrt{\xi^2 + \eta^2}$; quare unum signum (—) respiciendum. Sequitur igitur, tertiis potestatibus neglectis

$$13)\begin{cases}\frac{x_{\prime\prime}}{R} = \xi - \frac{a_\prime}{c_\prime}\frac{\varDelta}{R} - \frac{1}{2}\frac{a_\prime}{c_\prime}(\xi^2+\eta^2)\\ \frac{y_{\prime\prime}}{R} = \eta - \frac{b_\prime}{c_\prime}\frac{\varDelta}{R} - \frac{1}{2}\frac{b_\prime}{c_\prime}(\xi^2+\eta^2)\end{cases}$$

Hac igitur in approximatione unam aequationum radicem accepimus, quae in intervallo respiciendo sita est; eodem modo in sequenti approximatione res sese habet; e quibus sequitur, radicium aequationum propositarum aut unam in illo intervallo esse, aut, si plures in eo sunt, has tantum quantitatibus quarti ordinis discrepare; reliquas vero radices aut in aliis intervallis non respiciendis aut imaginarias esse. Demonstravi igitur, in quoque plani superioris puncto unum radium e lamella exire.

Ut vero accuratiorem valorem quantittatum $\frac{x_{\prime\prime}}{R}$ et $\frac{y_{\prime\prime}}{R}$ accipiam, pono

$$14)\begin{cases}\frac{x_{\prime\prime}}{R} = \xi - \frac{a_\prime}{c_\prime}\frac{\varDelta}{R} - \frac{1}{2}\frac{a_\prime}{c_\prime}(\xi^2+\eta^2) + \varepsilon\\ \frac{y_{\prime\prime}}{R} = \eta - \frac{b_\prime}{c_\prime}\frac{\varDelta}{R} - \frac{1}{2}\frac{b_\prime}{c_\prime}(\xi^2+\eta^2) + \varepsilon_\prime\end{cases}$$

Quos valores si introducimus in aequationes (12) et si tertias modo quantitatum ξ et η potestates respicimus, sequitur, cum $\frac{\varDelta}{R}$ ordine $\xi^2+\eta^2$:

$$15)\begin{cases}\varepsilon = \frac{a_\prime}{c_\prime}\frac{a_\prime\xi+b_\prime\eta}{c_\prime}\frac{\varDelta}{R} - \frac{1}{2}\frac{a_\prime}{c_\prime}\frac{a_\prime\xi+b_\prime\eta}{c_\prime}(\xi^2+\eta^2) - \xi(\xi^2+\eta^2)\\ \varepsilon_\prime = \frac{b_\prime}{c_\prime}\frac{a_\prime\xi+b_\prime\eta}{c_\prime}\frac{\varDelta}{R} - \frac{1}{2}\frac{b_\prime}{c_\prime}\frac{a_\prime\xi+b_\prime\eta}{c_\prime}(\xi^2+\eta^2) - \eta(\xi^2+\eta^2)\\ \text{denique}\\ \frac{z_{\prime\prime}}{R} = \frac{\varDelta}{R} + \frac{1}{2}(\xi^2+\eta^2) - \frac{a_\prime\xi+b_\prime\eta}{c_\prime}\frac{\varDelta}{R} - \frac{1}{2}\frac{a_\prime\xi+b_\prime\eta}{c_\prime}(\xi^2+\eta^2)\end{cases}$$

Tum consequitur ex aequationibus (10)

$$16)\begin{cases} m = -a_{,} - 2c_{,}\xi + 2\xi\,.\,(a_{,}\xi + b_{,}\eta) + a_{,}(\xi^2+\eta^2) \\ \quad - \frac{1-4c_{,}^2}{c_{,}}\xi(\xi^2+\eta^2) + 2a_{,}\frac{\Delta}{R} - 2\frac{\Delta}{R}\,\frac{(3a_{,}^2+b_{,}^2)\xi + 2a_{,}b_{,}\eta}{c_{,}} \\ n = -b_{,} - 2c_{,}\eta + 2\eta\,(a_{,}\xi + b_{,}\eta) + b_{,}(\xi^2+\eta^2) \\ \quad - \frac{1-4c_{,}^2}{c_{,}}\eta(\xi^2+\eta^2) + 2b_{,}\frac{\Delta}{R} - 2\frac{\Delta}{R}\,\frac{2a_{,}b_{,}\xi + (a_{,}^2+3b_{,}^2)\eta}{c_{,}} \\ o = c_{,} - 2(a_{,}\xi + b_{,}\eta) + \frac{1-3c_{,}^2}{c_{,}}(\xi^2+\eta^2) \\ \quad + \frac{1+4c_{,}^2}{c_{,}^2}(a_{,}\xi + b_{,}\eta)\xi^2) + 2\frac{a_{,}^2+b_{,}^2}{c_{,}}\frac{\Delta}{R} \\ \quad - 2\,\frac{1-3c_{,}^2}{c_{,}^2}(a_{,}\xi + b_{,}\eta)\frac{\Delta}{R}; \end{cases}$$

quae aequationes efficiunt.

$$m^2 + n^2 + o^2 = a_{,}^2 + b_{,} + c_{,}^2 = 1$$

usque ad membra quarti ordinis. — Atque m, n, o determinantia radii CD, radii vero DE determinantia (Fig. 1) sunt Nm, Nn, $\sqrt{1 - N^2(m^2+n^2)}$, cum radii DG, statim a superiore plano reflexi, determinantia sint $-a$, $-b$, $+c$; quare sequitur e formulis (16), radium DE cum DG non concidere, sed cum eo angulum quendam continere.

3. Radii componuntur secundum theoriam undulationis, et valor diametri annulorum evolvitur.

Jamjam sint componentes radii cujusdam polarisatis incidentis

$$17)\begin{cases} u = A\cos.\left\{\frac{t}{T} - \frac{a x + b y + c z}{\lambda}\right\} 2\pi \\ v = B\cos.\left\{\frac{t}{T} - \frac{a x + b y + c z}{\gamma}\right\} 2\pi \\ w = C\cos.\left\{\frac{t}{T} - \frac{a x + b y + c z}{\lambda}\right\} 2\pi; \end{cases}$$

qui radius fractus in superiore lamina est:

$$18)\begin{cases} u_{,} = A_{,}\cos.\left\{\frac{t}{T} - \frac{a_{,}x + b_{,}y + c_{,}z}{\lambda_{,}}\right\} 2\pi \\ v_{,} = B_{,}\cos.\left\{\frac{t}{T} - \frac{a_{,}x + b_{,}y + c_{,}z}{\lambda_{,}}\right\} 2\pi \\ w_{,} = C_{,}\cos.\left\{\frac{t}{T} - \frac{a_{,}x + b_{,}y + c_{,}z}{\lambda_{,}}\right\} 2\pi \end{cases}$$

ubi $a = Na_{,}$, $b = Nb_{,}$, $a_{,}^2 + b_{,}^2 + c_{,}^2 = 1$.

Atque hic radius cum sphaeram attingat in puncto $x_{,,}$, $y_{,,}$, $z_{,,}$, ut radii reflexi aequationes accipiam, transformatione nota utor:

$$
19) \left\{
\begin{array}{l}
x = x_{,,} + \alpha\, X + \alpha'\, Y + \alpha''\, Z \\
y = y_{,,} + \beta\, X + \beta'\, Y + \beta''\, Z \\
z = z_{,,} + \gamma\, X + \gamma'\, Y + \gamma''\, Z \\
\quad \text{et inverse} \\
X = \alpha\,(x - x_{,,}) + \beta\,(y - y_{,,}) + \gamma\,(z - z_{,,}) \\
Y = \alpha'\,(x - x_{,,}) + \beta'\,(y - y_{,,}) + \gamma'\,(z - z_{,,}) \\
Z = \alpha''\,(x - x_{,,}) + \beta''\,(y - y_{,,}) + \gamma''\,(z - z'')
\end{array}
\right.
$$

Axis Z sit parallela normali sphaerae in puncto $x_{,,}$, $y_{,,}$, $z_{,,}$; est igitur

$$\alpha'' = \frac{x_{,,}}{R},\ \beta'' = \frac{y_{,,}}{R},\ \gamma'' = \frac{z_{,,} - R - \Delta}{R}.$$

Inter reliquas vero sex transformationes coefficientes quinque aequationes valent, ut una ex iis arbitraria maneat; de qua ita statuo, ut axis Y perpendicularis plano incidentiae. Tum est:

$$
\begin{array}{l}
\alpha'\, x_{,,} + \beta'\, y_{,,} + \gamma'\,(z_{,,} - \Delta - R) = 0 \\
\alpha'\, a_{,} + \beta'\, b_{,} + \gamma'\, c_{,} = 0 \\
\alpha'\, \alpha' + \beta'\, \beta' + \gamma'\, \gamma' = 1 \\
\alpha\, \alpha' + \beta\, \beta' + \gamma\, \gamma' = 0 \\
\alpha\, x_{,,} + \beta\, y_{,,} + \gamma\,(z_{,,} - R - \Delta) = 0 \\
\qquad \alpha\alpha + \beta\beta + \gamma\gamma = 1
\end{array}
$$

E quibus aequationibus sequitur:

$$\alpha' = \frac{1}{D}\left\{c_{,}\frac{y_{,,}}{R} - b_{,}\frac{z_{,,} - \Delta - R}{R}\right\}$$

$$\beta' = \frac{1}{D}\left\{a_{,}\frac{z_{,,} - R - \Delta}{R} - c_{,}\frac{x_{,,}}{R}\right\}$$

$$\gamma' = \frac{1}{D}\left\{b_{,}\frac{x_{,,}}{R} - a_{,}\frac{y_{,,}}{R}\right\}$$

$$\alpha = \frac{1}{D_{,}}\left\{a_{,} - \frac{x_{,,}}{R}\left[a_{,}\frac{x_{,,}}{R} + b_{,}\frac{y_{,,}}{R} + c_{,}\frac{z_{,,} - \Delta - R}{R}\right]\right\}$$

$$\beta = \frac{1}{D_{,}}\left\{b_{,} - \frac{y_{,,}}{R}\left[a_{,}\frac{x_{,,}}{R} + b_{,}\frac{y_{,,}}{R} + c_{,}\frac{z_{,,} - \Delta - R}{R}\right]\right\}$$

$$\gamma = \frac{1}{D_{,}}\left\{c_{,} - \frac{z_{,,} - \Delta - R}{R}\left[a_{,}\frac{x_{,,}}{R} + b_{,}\frac{y_{,,}}{R} + c_{,}\frac{z_{,,} - \Delta - R}{R}\right]\right\}$$

$$D = D_{,} = \pm\sqrt{1 - \left[a_{,}\frac{x_{,,}}{R} + b_{,}\frac{y_{,,}}{R} + c_{,}\frac{y_{,,} - \Delta - R}{R}\right]^2}$$

Quare efficitur

$$a_, x + b_, y + c_, z = a_, x_{,,} + b_, y_{,,} + c_, z_{,,} \pm X \sin \psi + Z \cos \psi,$$

ubi

$$\cos \psi = a_, \frac{x_{,,}}{R} + b_, \frac{y_{,,}}{R} + c_, \frac{z_{,,} - \varDelta - R}{R}$$

Componentes igitur radii (18) secundum novas axes sunt

$$20) \begin{cases} U = \varDelta \cos \left\{ \frac{t}{T} - \frac{a_, x_{,,} + b_, y_{,,} + c_, z_{,,} \pm X_, \sin \psi + Z_, \cos \psi}{\lambda_,} \right\} 2\pi \\ V = B \cos \left\{ \frac{t}{T} - \frac{a_, x_{,,} + b_, y_{,,} + c_, z_{,,} \pm X \sin \psi + Z_, \cos \psi}{\lambda_,} \right\} 2\pi \\ W = \varGamma \cos \left\{ \frac{t}{T} - \frac{a_, x_{,,} + b_, y_{,,} + c_, z_{,,} \pm X \sin \psi + Z \cos \psi}{\lambda_,} \right\} 2\pi \end{cases}$$

ubi

$$\begin{aligned} \varDelta &= A_, \alpha + B_, \beta + C_, \gamma \\ B &= A_, \alpha' + B_, \beta' + C_, \gamma' \\ \varGamma &= A_, \alpha'' + B_, \beta'' + C_, \gamma'' \end{aligned}$$

Radius vero reflexus in sphaera est:

$$21) \begin{cases} U' = \varDelta_, \cos \left\{ \frac{t}{T} - \frac{a_, x_{,,} + b_, y_{,,} + c_, z_{,,} \pm X \sin \psi - Z \cos \psi}{\lambda_,} \right\} 2\pi \\ V' = B_, \cos \left\{ \frac{t}{T} - \frac{a_, x_{,,} + b_, y_{,,} + c_, z_{,,} \pm X \sin \psi - Z \cos \psi}{\lambda_,} \right\} 2\pi \\ W' = \varGamma_, \cos \left\{ \frac{t}{T} - \frac{a_, x_{,,} + b_, y_{,,} + c_, z_{,,} \pm X \sin \psi - Z \cos \psi}{\lambda_,} \right\} 2\pi \end{cases}$$

$$\varDelta_, = \varDelta \frac{\sin(\psi - \psi_,)}{\sin(\psi + \psi_,)}, \quad \varGamma_, = \varGamma \frac{\sin(\psi - \psi_,)}{\sin(\psi - \psi_,)},$$

$$B_, = B \frac{\operatorname{tg}(\psi - \psi_,)}{\operatorname{tg}(\psi + \psi_,)}, \quad \sin \psi_, = M \sin \psi$$

ubi M exponens refractionis ab inferiore medio in lamellam.

Jam si valores (21) retransformes in veteres coordinatas, efficitur:

$$22) \begin{cases} u'' = A_2 \cos \\ \quad \left\{ \frac{t}{T} - \frac{(a_, + m) x_{,,} + (b_, + n) y_{,,} + (c_, + o) z_{,,} - (mx + ny + oz)}{\lambda_,} \right\} 2\pi \\ v'' = B_2 \cos \\ \quad \left\{ \frac{t}{T} - \frac{(a_, + m) x_{,,} + (b_, + n) y_{,,} + (c_, + o) z_{,,} - (mx + ny + oz)}{\lambda_,} \right\} 2\pi \\ w'' = C_2 \cos \\ \quad \left\{ \frac{t}{T} - \frac{(a_, + m) x_{,,} + (b_, + n) y_{,,} + (c_, + o) z_{,,} - (mx + ny + oz)}{\lambda_,} \right\} 2\pi \end{cases}$$

ubi $$A_2 = A_, \alpha + B_, \alpha' + \Gamma_, . \alpha''$$
etc.

Hic vero radius in puncto $x_{,,,}$, $y_{,,,}$, $z_{,,,} = O$ in superius medium refringitur atque est radius fractus:

$$23)\ \begin{cases} u''' = A_3 \cos \\ \left\{ \frac{t}{T} - \frac{N(a_, + m)x_{,,} + N(b_, + n)y_{,,} + N(c_, + o)z_{,,} - (Nmx + Nny + o'z)}{\lambda} \right\} 2\pi \end{cases}$$
etc.

$$o' = \sqrt{1 - N^2 (m^2 + n^2)}$$

Sunt igitur occillationes, quas exercet punctum $x_{,,,}$, $y_{,,,}$, $z_{,,,} = O$ hujus impulsus causa:

$$\begin{cases} u_0''' = A_3 \cos \\ \left\{ \frac{t}{T} - \frac{N(a_, + m)x_{,,} + N(b_, + n)y_{,,} + N(c_, + o)z_{,,} - (Nmx_{,,,} + Nny_{,,,})}{\lambda} \right\} 2\pi \end{cases}$$
etc.

Atque si quantitates $x_{,,}$, $y_{,,}$, $z_{,,}$, m, n, o exprimuntur per $\xi = Rx_{,,,}$, $\eta = Ry_{,,,}$, consequitur:

$$24)\ \begin{cases} u_0''' = A_3 \cos \left\{ \frac{t}{R} - \frac{a\xi + b\eta + \delta}{\lambda} R \right\} 2\pi \\ v_0''' = B_3 \cos \left\{ \frac{t}{T} - \frac{a\xi + b\eta + \delta}{\lambda} R \right\} 2\pi \\ w_0''' = C_3 \cos \left\{ \frac{t}{T} - \frac{a\xi + b\eta + \delta}{\lambda} R \right\} 2\pi \end{cases}$$

$$\delta = \sqrt{N^2 - 1 + c^2}\,(\xi^2 + \eta^2) - (a\xi + b\eta)(\xi^2 + \eta^2)$$
$$+ 2\frac{\Delta}{R}\sqrt{N^2 - 1 + c^2} - 2\,(a\xi + b\eta)\frac{\Delta}{R}$$
$$(\sqrt{N^2 - 1 + c^2} = Nc_,)$$

In ipsa vero superiore lamina in puncto $x_{,,,}$, $y_{,,,}$, etiam e fasci radiorum incidentium radius quidam reflectitur; atque hic radius reflexus exercet in punctum $x_{,,,}$, $y_{,,,}$ hunc impulsum:

$$25)\ \begin{cases} u_, = A' \cos \left\{ \frac{t}{T} - \frac{a\xi + b\eta}{\lambda} R \right\} 2\pi \\ v_, = B' \cos \left\{ \frac{t}{T} - \frac{a\xi + b\eta}{\lambda} R \right\} 2\pi \\ w_, = C' \cos \left\{ \frac{t}{T} - \frac{a\xi + b\eta}{\lambda} R \right\} 2\pi \end{cases}$$

Atque duo impulsus u''' et $u_,$ componuntur ad undulationem linearem, eodem modo $v_,$ et v''', $w_,$ et w'''. Hae tres vero lineares undulationes componuntur ad ellipticum puncti $x_{,,,}$, $y_{,,,}$ motum, cujus intensitas:

$$26)\ J = 2\left(\frac{\pi}{T}\right)^2 \Big\{A_3^2 + B_3^2 + C_3^2 + A'^2 + B'^2 + C'^2 + 2\,(A'A_3 + B'B_3 + C'C_3)\cos\frac{R\,2\pi}{\lambda}\,\delta\Big\}$$

Sunt vero A', B', C' constantes, A_3, B_3, C_3 autem functiones quantitatum ξ et η; et cum, sicuti experimentum docet, parvi modo quantitatum ξ et η valores agantur, prima certe approximatio efficietur, si interdum quantitatibus A_3, B_3, C_3 sicuti constantibus utemur; quae quidem approximatio eo diligentior erit, quia oculus parvas intensitatis varietates vix percipere potest.

Jam est dijudicandum, quo signo utatur membrum ultimum formulae 26). Quam ad rem quantitatem

$$A'A_3 + B'B_3 + C'C_3$$

explicatam cogito secundum potestates quantitatum ξ et η; et cum ξ et η tam parvae quantitates sint, membrum constans explicationis maxime praecipuum totius expressionis erit, itaque solum signum expressionis constituet. Membrum vero constans accipies, si pro sphaera planum superioris plani parallelum ponas. Tum vero efficitur, luce incidenti in plano incidentiae polarisata:

$$A'A_3 + B'B_3 + C'C_3 = \frac{2\sin\varphi\cos\varphi}{\sin(\varphi+\varphi_{,})}\ \frac{\sin(\varphi_{,}-\varphi_{,,})}{\sin(\varphi_{,}+\varphi_{,,})}\ \frac{2\sin\varphi_{,}\cos\varphi_{,}}{\sin(\varphi+\varphi_{,})}\ \frac{\sin(\varphi-\varphi_{,})}{\sin(\varphi+\varphi_{,})}$$

ubi $\quad c = \cos\varphi,\quad c_{,} = \cos\varphi_{,},\quad \sin\varphi_{,,} = M\sin\varphi_{,}$.

E quibus sequitur, si quantitatum M et N aut utraque > 1 aut utraque < 1, $A'A_3 +$ etc. esse negativum; si autem $M > 1$, $N < 1$ aut $M < 1$, $N > 1$, $A'A_3 + \ldots$ esse positivum; eodemque modo lux polarisata perpendicularitur plano incidentiae sese habet. Quare primo in casu J est maximum, ubi:

$$27)\ \begin{cases} \dfrac{R\,2\pi}{\lambda}\,\delta = (2h+1)\,\pi \\ J \text{ est minimum, ubi} \\ \dfrac{R\,2\pi}{\lambda}\,\delta = 2h\,\pi; \end{cases}$$

altero vero casu maximi et minimi valores permutandi.

Atque si in his aequationibus jam tertii ordinis membra prae secundi ordinis membro negligere velis, efficiatur:

$$28)\ \varrho^2 = \xi^2 + \eta^2 = \frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R} - 2\frac{\Delta}{R},$$

quae eadem formula est, quae ex vetere theoria apparuit. Jam quum quantitatis $\varrho = \sqrt{\xi^2 + \eta^2}$ ubique parvi valores agantur, apparet, hanc sane primam approximationem, re vera igitur esse

$$29)\quad \begin{cases} \xi = \varrho \cos \vartheta = \sqrt{\frac{2h+1}{2\,Nc_{,}}\frac{\lambda}{R} - 2\frac{\Delta}{R}} \cos \vartheta + \varepsilon \\ \eta = \varrho \sin \vartheta = \sqrt{\frac{2h+1}{2\,Nc_{,}}\frac{\lambda}{R} - 2\frac{\Delta}{R}} \sin \vartheta + \varepsilon_{,} \end{cases}$$

Qui valores si in aequationem 27) introducuntur, facile intelligas, quantitates ε et $\varepsilon_{,}$ esse minoris ordinis qaum $\sqrt{\frac{\lambda}{R}}$, certe ordinis $\left|\sqrt{\frac{\lambda}{R}}\right|^2$. Quare si valores quantitatum ξ et η non accuratiores quam ad quantitates ordinis $\frac{\lambda}{R}$ computare velis, membra $\varepsilon\,\frac{\lambda}{R}$, ε^2 negligenda sunt prae $\varepsilon \sqrt{\frac{\lambda}{R}}$; et consequitur

$$\varepsilon = \frac{1}{2}\,\frac{a_{,}}{c_{,}}\,\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R},\quad \varepsilon_{,} = \frac{1}{2}\,\frac{b_{,}}{c_{,}}\,\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R},$$

quare

$$30)\quad \begin{cases} \xi = \sqrt{\frac{2h+1}{2\,Nc_{,}}\frac{\lambda}{R} - 2\frac{\Delta}{R}} \cos \vartheta + \frac{1}{2}\,\frac{a_{,}}{c_{,}}\,\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R} \\ \eta = \sqrt{\frac{2h+1}{2\,Nc_{,}}\frac{\lambda}{R} - 2\frac{\Delta}{R}} \sin \vartheta + \frac{1}{2}\,\frac{b_{,}}{c_{,}}\,\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R} \\ \sqrt{\xi^2+\eta^2} = \sqrt{\frac{2h+1}{2\,Nc_{,}}\frac{\lambda}{R} - 2\frac{\Delta}{R}} \\ \qquad + \frac{1}{2}\,\frac{a_{,}\cos\vartheta + b_{,}\sin\vartheta}{c_{,}}\,\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R}. \end{cases}$$

Jam est demonstrandum, membra neglecta in nostra quidem approximatione nullo momento esse. Sunt enim in quantitatibus $a_{,} + m$, $b_{,} + n$, $c_{,} + o$, $\frac{x_{,,}}{R}$, $\frac{y_{,,}}{R}$, $\frac{z_{,,}}{R}$ membra tertii ordinis quantitatum ξ et η plane accurata; membra igitur neglecta nullius momenti sunt ad membra tertii ordinis quantitatum ξ et η, sed summum ad membra quarti ordinis. Si vero etiam quarti ordinis membrum respiciamus, pro $\Delta = o$, $\xi = \varrho \cos \vartheta$, $\eta = \varrho \sin \vartheta$ aequatio sequatur hujus formae:

$$a)\quad \varrho^2 - A\varrho^3 + B\varrho^4 = \frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R}$$

Atque cum ϱ sit

$$\varrho = \sqrt{\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R}} + \varepsilon,\ \text{est}$$

$$\varepsilon = \frac{A\left|\sqrt{\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R}}\right|^{3} - B\left|\sqrt{\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R}}\right|^{4}}{2\left|\sqrt{\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R}}\right| - 3A\left|\sqrt{\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R}}\right|^{2} + 4B\left|\sqrt{\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R}}\right|^{3}}$$

Jam si evolvitur ε secundum potestates $\sqrt{\frac{\lambda}{R}}$ et $\sqrt{\frac{\lambda}{R}}^{3}$ neligitur, patet:

$$\varepsilon = \frac{1}{2}\,A\,\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R},$$

nunc si ponitur

$$\varrho = \sqrt{\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R}} + \frac{1}{2}\,A\,\frac{2h+1}{2\,Nc_{,}}\,\frac{\lambda}{R} + \varepsilon_{,}$$

et hic valor in aequationem a) introducitur, valor quantitatis $\varepsilon_{,}$ sequitur ordine $\sqrt{\frac{\lambda}{R}}^{3}$ E quibus consequitur, valores quantitatum ξ, η, ϱ plane accuratas esse, si $\sqrt{\frac{\lambda}{R}}^{3}$ negligitur prae $\sqrt{\frac{\lambda}{R}}$ et $\sqrt{\frac{\lambda}{R}}^{2}$. Hoc autem membram nullo omnino momento esse, ita patet: pro linea D spectri est $\lambda = 0{,}0005888^{\text{mm}}$, R ponatur $= 100^{\text{mm}}$, qui valor tam parvus, ut annuli vix nudis oculis cernas, tum est membrum $\sqrt{\frac{\lambda}{R}}^{3}$ modo 0,00000588 primi ordinis. Membrum igitur $\sqrt{\frac{\lambda}{R}}^{3}$ merito plane neglectum est.

Explicatio, quam dedimus ad hoc, valet pro luce ad libitum polarisata, quare etiam pro luce naturali.

4. Aequationes (30) discutiuntur.

In aequationibus (30) pono

$$\xi = R\,.\,x_{,,,,}\quad \eta = R\,.\,y_{,,,,}$$
$$a = \sin\varphi,\ b = o,\ c = \cos\varphi$$
$$a_{,} = \sin\varphi_{,,}\ b = o,\ c_{,} = \cos\varphi_{,}.$$

Tum est

$$31)\begin{cases} x_{,,,} = \sqrt{\dfrac{2h+1}{2N\cos\varphi_,}\lambda - \varDelta\sqrt{R}}\cos\vartheta \\ \qquad + \dfrac{1}{2}\, tg\,\varphi_,\, \dfrac{2h+1}{2N\cos\varphi_,}\lambda \\ y_{,,,} = \sqrt{\dfrac{2h+1}{2N\cos\varphi_,}\lambda - \varDelta\sqrt{R}}\sin\vartheta, \end{cases}$$

e puibus sequitur:

$$32)\ \left\{x_{,,,} - \frac{1}{2}\, tg\,\varphi_,\, \frac{2h+1}{2N\cos\varphi_,}\lambda\right\}^2 + y_{,,,}^{\,2} = \left\{\frac{2h+1}{2N\cos\varphi_,}\lambda - \varDelta\right\} R$$

Ea igitur laminae planae puncta, quae pari intensitate utuntur, in circulo quodam sita sunt, cujus tamen centrum pro $\varDelta = o$ non est punctum contactus plani cum sphaera, sed cujus centrum in plano incidentiae quantitate $\frac{1}{2}\, tg\,\varphi_,\, \frac{2h+1}{2N\cos\varphi_,}\lambda$ dilatum est. Secundum ea igitur, quae diximus, annuli etiam circulos efficiunt, sed non amplius, sicuti prior methodus docuit, circulos concentricos sed excentricos; atque distantia omnium centrorum a puncto contactus varia est. Radius vero circulorum pro $\varDelta = o$ eodem valore est, quo in pristina methodo, sed radius vector a puncto contactus ad aliquem circulum ductus alio valore est, hoc enim

$$r = \sqrt{\frac{2h+1}{2N\cos\varphi_,}\lambda - 2\varDelta\sqrt{R}} + \frac{1}{2}\, tg\,\varphi_,\, \frac{2h+1}{2N\cos\varphi_,}\lambda\cos\vartheta;$$

ubi ϑ angulus est, quem radius vectur continet cum plano incidentiae. Quare pro $\vartheta = \frac{\pi}{2}$, id est pro plano perpendiculari ad planum incidentiae idem radii vectoris valor patet qui prius; pro $\vartheta = o$ vero radius vector membro quodam correctionali deditus est, quod membrum contrario valore est pro $\vartheta = \pi$, ut differentia inter utrumque valorem sit par duplici membro correctionali, id est

$$= tg\,\varphi_,\, \frac{2h+1}{2N\cos\varphi_,}\lambda.$$

Porro apparet, radium vectorem perpendicularem plano incidentiae legem Newtonianam sequi, id est, hunc radium vectorem pro variis annulis claris in ratione esse radicium quadraticarum numeris imparibus, pro annulis obscuris in ratione radicium quadraticarum e numeris paribus; pro radio autem vectore in plano incidentiae posito hanc legem non amplius valere.

Accedit, ut etiam alia res respicienda sit. Ea enim puncta, quae pari intensitate utuntur, sunt re vera in circulo quodam; hujus circuli autem in planitiem cum oblique lux incidat, planities est projicienda in planum perpendiculare radio lucis, ut vera imago in oculum perveniens accipiatur. E quibus apparet, annulos videri ut ellipses, quarum minor axis in plano incidentiae, atque ellipses excentricas. Efficitur igitur

$$33)\left\{\begin{aligned} x_{,,,} &= \sqrt{\frac{2h+1}{2N\cos\varphi_{,}}\lambda - \Delta\sqrt{R}}\cos\vartheta\cos\varphi \\ &\quad + \frac{1}{2}\,tg\,\varphi_{,}\cos\varphi\,\frac{2h+1}{2N\cos\varphi_{,}}\,\lambda \\ y_{,,,} &= \sqrt{\frac{2h+1}{2N\cos\varphi_{,}}\lambda - \Delta\sqrt{R}}\sin\vartheta. \end{aligned}\right.$$

5. Ultima refractio respicitur.

Sicuti jam pag. 7 annotavi, is Casus, cui adhuc operam dedi, mere idealis est; in quo ponitur (Fig. 1), duos radios, qui a quoque puncto superioris superficiei lamellae exeant, nullo modo mutatos in oculum pervenire, id est, oculum esse in ipso superiore medio. Qui casus re vera nunquam fere occurrit, sed, cum oculus non ipso in superiore medio, uterque radius a puncto D exiens etiam refrangitur, qua re phasium differentia mutatur. Sit jam (Fig. 3) medium superius tabula, duobus planis parallelis finita crassitudine d; simul simplicitatis causa pono, medium in lamella esse ejusdem naturae cujus id medium, in quo oculus est. A puncto quodam D in superiore lamellae limite posito exeunt duo radii, quorum alter DE in ipso plano, alter $DE_{,}$ in sphaera reflexus est. Atque radius DE in puncto E in aërem introit versus EF, alter radius in $E_{,}$ versus $E_{,}F_{,}$. Erant vero determinantia radiorum incidentium in superiore lamellae limite, hoc loco igitur in vitro, a, b, c. N est igitur exponens refractionis a vitro in aërem, N igitur < 1; $a_{,,}$ $b_{,,}$ $c_{,,}$ determinantia sunt radiorum parallelorum in aëre. Quibus positis erat radius

$$CD)\quad A\cos\left\{\frac{t}{T} - \frac{ax+by+cz}{\lambda}\right\}2\pi$$

ubi λ longitudo undulationis in vitro, $\lambda_{,}$ in aëre; radius autem reflexus

$$DE)\quad A_{,} \cos\left\{\frac{t}{T} - \frac{ax+by-cz}{\lambda}\right\} 2\,\pi$$

$$= A_{,} \cos\left\{\frac{t}{T} - \frac{ax+by-c\,(z+d)+cd}{\lambda}\right\} 2\,\pi$$

Qui radius cum in planitie $z + d = O$ refringatur in aërem, est radius refractus

$$EF)\quad A_2 \cos\left\{\frac{t}{T} - \frac{a_{,}x+b_{,}y-c_{,}\,(z+d)}{\lambda_{,}} - \frac{cd}{\lambda}\right\} 2\,\pi$$

$$= A_2 \cos\left\{\frac{t}{T} - \frac{a_{,}x+b_{,}y-c_{,}\,(z+d)+\frac{1}{N}\,c\,d}{\lambda_{,}}\right\} 2\,\pi$$

cum $N = \frac{\lambda}{\lambda_{,}}$; aut si ponas

$$d_{,} = d\left\{1 - \frac{1}{N}\,\frac{c}{c_{,}}\right\}$$

est radius EF:

$$34)\quad A_2 \cos\left\{\frac{t}{T} - \frac{a_{,}x+b_{,}y-c_{,}\,(z+d_{,})}{\lambda_{,}}\right\} 2\,\pi;$$

id est radius EF eadem aequatione utitur, qua radius reflexus in G (Fig. 4) in planitie $z + d_{,} = O$, qui tamen sine refractione in oculum pervenit; eodemquae modo omnes radii EF sese habent. Jam vero radii $DE_{,}$ semel in sphaera in puncto $x_{,,,}$ $y_{,,,}$ $z_{,,}$ reflexi erat aequatio

$$A_3 \cos\left\{\frac{t}{T} - \frac{N(a_{,}+m)x_{,,}+N(b_{,}+n)\,y_{,,}+N(c_{,}+o)\,z_{,,}-(Nmx+Nny+o'z)}{\lambda}\right\} 2\,\pi$$

ubi $o' = \sqrt{1 - N^2 + N^2\,o^2}$.

Qui radius si in plano $z + d = o$ refringitur in aërem, radii fracti $E_{,}\ F_{,}$ aequatio est

$$35)\quad A_3 \cos \left\{\frac{t}{T} - \frac{(a_{,}+m)\,x_{,,}+(b_{,}+n)\,y_{,,}+(c_{,}+o)\,z_{,,}-(mx+ny)-o(z+d_{,}')}{\lambda_{,}}\right\} 2\,\pi$$

ubi $$d_{,}' = d\left\{1 - \frac{1}{N}\,\frac{o'}{o}\right\}.$$

Jam duo radiorum sytemata sunt proposita, alterum radiorum EF, alterum radiorum $E_{,}F_{,}$; utrumque systema, aequationibus 34) et 35) expressum sicuti infractum in oculum pervenit; quare ipsa consideratio superior etiam hoc loco applicanda, si modo loco radiorum 25) ponimus systema 34), loco radiorum 24) systema 35); simul vero pro plano $z = o$ hoc loco est planum ponendum $z + d_{,} = o$. Sicut igitur antea duos radios in puncto aliquo plani $z = o$ decussantes composui, ita hoc loco duo radii in puncto plani $z + d_, = o$ decussantes componendi sunt. Quem ad finem aliquo puncto G plani $z + d_, = o$ constituto inquiro, qui radiorum 35) hoc punctum contingat, atque hunc radium exprimo coordinatis puncti G; denique vero hunc radium compono cum eo radio ex systemate 34), qui idem punctum G secat.

Atque primum ostendam, duos radios DE et $D_,E_{,,}$ qui ex eodem puncto D exeant, fractos et retro prorogatos non idem punctum G secare. Sint enim, ut antea, coordinatae puncti D $x = R\xi$, $y = R\eta$, sunt

$$DE) \quad -az = c(x - R\xi) \qquad DE) \quad Nmz = o'(x - R\xi)$$
$$-bz = c(y - R\eta) \qquad Nnz = o'(y - R\eta).$$

Punctorum E et $E_,$ coordinatae sunt

$$E) \quad x = R\,\xi + \frac{a}{c}\,d \qquad E_,) \quad x = R\,\xi - \frac{Nm}{o'}\,d$$
$$y = R\,\eta + \frac{b}{c}\,d \qquad y = R\,\eta - \frac{Nn}{o'}\,d,$$

unde radiorum fractorum aequationes sequuntur.

$$E\,F) - a_, (z + d) = c_, \left\{x - R\,\xi - \frac{a}{c}\,d\right\}$$
$$- b_, (y + d) = c_, \left\{y - R\,\eta - \frac{b}{c}\,d\right\}$$
$$E_, F_,) \; m\;(z + d) = o \left\{x - R\,\xi + \frac{Nm}{o_,}\,d\right\}$$
$$n\;(z + d) = o \left\{y - R\,\eta + \frac{Nn}{o'}\,d\right\}$$

Si vero duos radios in eodem puncto G plani $z + d_, = O$ convenire oporteret, pro $z = -d$, quantitates x et y in duobus systematis eadem esse deberent; esset igitur

$$\left(\frac{a}{c} + \frac{Nm}{o_,}\right) d = \left(\frac{a_,}{c_,} + \frac{m}{o}\right) \frac{d}{N}\;\frac{c}{c_,}$$
$$\left(\frac{b}{c} + \frac{Nn}{o_,}\right) d = \left(\frac{b_,}{c_,} + \frac{n}{o}\right) \frac{d}{N}\;\frac{c}{c_,}$$

aut, $\frac{Nm}{o_,}$, $\frac{m}{o}$ etc. secundum potestates quantitatum ξ et η evolutis et jam secundis potestatibus neglectis, esse deberet.

$$\frac{N^2 c_,^2}{c^2} \left\{\xi\left(1 + \frac{a^2}{c^2}\right) + \eta\,\frac{ab}{c^2}\right\} = \xi\left(1 + \frac{a_,^2}{c_,^2}\right) + \eta\,\frac{a_,b_,}{c_,^2}$$
$$\frac{N^2 c_,^2}{c^2} \left\{\xi\,\frac{ab}{c^2} + \eta\left(1 + \frac{b^2}{c^2}\right)\right\} = \xi\,\frac{a_,b_,}{c_,^2} + \eta\left(1 + \frac{b_,^2}{c_,^2}\right),$$

quibus aequationibus satis fieri non potest, cum sit $a = Na_{,,}$ $b = Nb_{,}$. Duo igitur radii, qui in aliquo puncto G conveniunt, non ab uno puncto D plani $z = O$ exierunt. —

Sint jam dati puncti G coordinatae $x, y, z = -d_{,,}$ radius ex systemate (35), hoc punctum contingens, exierit e puncto D: $x = R.\ \xi$, $y = R.\ \eta$ $z = o$; est:

$$x = R\,\xi - \frac{Nm}{o'}\, d + \frac{m}{o}\,\frac{1}{N}\,\frac{c}{c_{,}}\, d$$

$$y = R\,\eta - \frac{Nn}{o'}\, d + \frac{n}{o}\,\frac{1}{N}\,\frac{c}{c_{,}}\, d$$

Quarum aequationum auxilio, si ξ et η exprimuntur functiones quantitatum x et y, patet,

$$(36)\quad\left\{\begin{aligned}
\xi &= \frac{x_{,}}{R'} + \frac{1-N^2}{N^2 c_{,}^2}\,\frac{d}{R}\left\{\frac{2Nc_{,}}{c}\,\frac{x_{,}}{R} + \right.\\
&\quad\left. 2\,\frac{a}{c}\,\frac{2c^2+N^2-1}{Nc_{,}c^2}\left(a_{,}\,\frac{x_{,}}{R} + b_{,}\,\frac{y_{,}}{R}\right)\right\}\\
\eta &= \frac{y_{,}}{R} + \frac{1-N^2}{N^2 c_{,}^2}\,\frac{d}{R}\left\{\frac{2Nc_{,}}{c}\,\frac{y_{,}}{R} + \right.\\
&\quad\left. 2\,\frac{b}{c}\,\frac{2c^2+N^2-1}{Nc_{,}c^2}\left(a_{,}\,\frac{x_{,}}{R} + b_{,}\,\frac{y_{,}}{R}\right)\right\}\\
\frac{x_{,}}{R} &= \frac{x}{R} + \frac{a}{c}\,\frac{1-N^2}{N^2 c_{,}^2}\,\frac{d}{R};\quad \frac{y_{,}}{R} = \frac{y}{R} + \frac{b}{c}\,\frac{1-N^2}{N^2 c_{,}^2}\,\frac{d}{R}
\end{aligned}\right.$$

Atque in hac explicatione posui, quantitatem $\frac{d}{R}$ eodem esse ordine quo $\frac{x}{R}$, $\frac{y}{R}$, cui conditioni plerisque in casibus satis fit.

Est vero impulsus, quem radius (35) in punctum G exercet $u_{,} = A_{,} \cos$

$$\left\{\frac{t}{T} - \frac{(a_{,}+m)\,x_{,,} + (b_{,}+n)\,y_{,,} + (c_{,}+o)z_{,,} - (mx+ny) - o(d_{,}'-d_{,})}{\lambda_{,}}\right\} 2\pi$$

atque si in hac aequatione omnes quantitates per x et y exprimuntur, consequitur:

$$(37)\quad\left\{\begin{aligned}
u_{,} &= A_{,} \cos \left\{\frac{t}{T} - \frac{a_{,}x + b_{,}\,y + \delta}{\lambda_{,}}\right\} 2\pi\\
R.\ \delta &= 2c_{,}\,\frac{\Delta}{R} + c_{,}\,\frac{x_{,}^2+y_{,}^2}{R^2} - \left(a_{,}\,\frac{x_{,}}{R} + b_{,}\,\frac{y_{,}}{R}\right)\frac{x_{,}^2+y_{,}^2}{R^2}\\
&\quad + 2\,\frac{1-N^2}{Nc}\,\frac{d}{R}\,\frac{x_{,}^2+y_{,}^2}{R^2} - 2\,\frac{\Delta}{R}\left(a_{,}\,\frac{x_{,}}{R} + b_{,}\,\frac{y_{,}}{R}\right)\\
&\quad + 2\,\frac{1-N^2}{Nc_{,}^2}\,\frac{c^2+N^2c_{,}^2}{c^3}\,\frac{d}{R}\left(a_{,}\,\frac{x_{,}}{R} + b_{,}\,\frac{y_{,}}{R}\right)^2
\end{aligned}\right.$$

Cum is impulsus, quem radius (34) in punitum G exercet, sit:

$$u_0 = A \cos \left\{ \frac{t}{T} - \frac{a_{,} x + b_{,} y}{\lambda} R \right\} 2\pi$$

Qui duo impulsus si eodem modo componuntur atque antea pag. 20, consequitur, intensitatem undulationis puncti G esse maximum ubi $\frac{\delta}{\lambda_{,}} 2\pi = (2h + 1)\pi$

minimum, ubi $\frac{\delta_{,}}{\lambda_{,}} 2\pi = 2h\pi$

Eaedemque aequationes valent, sie etiam componentes secundum axes y et z respiciuntur. Unde consequitur, $a_{,} = \sin\varphi_{,}$ $b_{,} = O$ $c_{,} = \cos\varphi_{,}$ positis, pro maximis esse:

$$\begin{aligned} 38)\ \sqrt{x_{,}^2 + y_{,}^2} = & \sqrt{\frac{2h+1}{2\cos\varphi_{,}} \lambda_{,} - 2\Delta} \sqrt{R} \\ & + \frac{1}{2} tg\varphi_{,} \cos\vartheta \frac{2h+1}{2\cos\varphi_{,}} \lambda_{,} \\ & - \frac{1 - N^2}{N\cos\varphi \cos\varphi_{,}} d \sqrt{\frac{2h+1}{2\cos\varphi_{,}} \frac{\lambda_{,}}{R} - 2\frac{\Delta}{R}} \\ & \left\{ 1 + \frac{\cos^2\varphi + N^2\cos^2\varphi_{,}}{\cos^2\varphi \cos^2\varphi_{,}} tg^2\varphi_{,} \cos^2\vartheta \right\} \end{aligned}$$

ubi ϑ angulus, quem radius vector annulorum continet cum plano incidentiae. Atque pro minimis eadem aequatio valet, si modo quantitatis $2h + 1$ loco ponitur $2h$. Ex aequatione vero (38) consequitur:

1) centrum annnulorum esse in puncto

$$x_{,} = O \quad y_{,} = o, \text{ id est in puncto}$$

$$x = - tg\varphi \frac{1 - N^2}{N^2 \cos^2\varphi} d, \; y = O;$$

centrum igitur non amplius est punctum contactus sphaerae cum plano, sed annuli communi dilatatatione utuntur in plano incidentiae.

2) Aequatio (38) non amplius tam simplicem interpretationem admittit, quam aequatio (32); ubi pones $\cos\vartheta = \frac{x_{,}}{\sqrt{x_{,}^2 + y_{,}^2}}$, accipies aequationem ejus curvae, quam annuli efficiunt. Et cum membrum primum dextrae partis magnitudine alia multo superet, etiam nunc annulorum forma simillima erit circulis sive ob projectiooem ellipsibus excentricis.

Equidem formam curvae non diligentius inquiram, sed intendam ad computandos annulorum radios vectores. Atque sequitur $r = \sqrt{x_{,}^2 + y^2}$ posito, et pro $\vartheta = \frac{\pi}{2}$ et pro $\vartheta = \frac{3\pi}{2}$ esse:

$$9)\ r = \sqrt{\frac{2h+1}{2\cos\varphi_{,}}\lambda_{,} - 2\varDelta}\ \sqrt{R}$$
$$- \frac{1-N^2}{N\cos\varphi\cos\varphi_{,}}\ d\ \sqrt{\frac{2h+1}{2\cos\varphi_{,}}\frac{\lambda_{,}}{R} - 2\frac{\varDelta}{R}}$$

Duo igitur radii vectores, perpendiculares plano incidentiae, pares sunt; annuli igitur symmetrice dividuntur plano incidentiae.

Porro, si consideres, planum annulorum esse projiciendum in id planum, quod perpendiculare radio lucis, quare radium vectorem in plano incidentiae ductum esse multiplicandum cum $\cos\varphi_{,}$, radio hoc plano perpendiculari immutato menente; efficiatur pro $\vartheta = O$:

$$40)\ r_{,} = \cos\varphi_{,} \sqrt{\frac{2h+1}{2\cos\varphi_{,}}\lambda_{,} - 2\varDelta}\ \sqrt{R} + \frac{1}{2}\sin\varphi_{,}\frac{2h+1}{2\cos\varphi_{,}}\lambda_{,}$$
$$-\frac{1-N^2}{N\cos\varphi} d \sqrt{\frac{2h+1}{2\cos\varphi_{,}}\frac{\lambda_{,}}{R} - 2\frac{\varDelta}{R}} \left\{1 + \frac{\cos^2\varphi + N^2\cos^2\varphi_{,}}{\cos^2\varphi\cos^2\varphi_{,}} tg^2\varphi_{,}\right\},$$

pro $\vartheta = \pi$

$$41)\ r_{,,} = \cos\varphi_{,} \sqrt{\frac{2h+1}{2\cos\varphi_{,}}\lambda_{,} - 2\varDelta}\ \sqrt{R} - \frac{1}{2}\sin\varphi_{,}\frac{2h+1}{2\cos\varphi_{,}}\lambda_{,}$$
$$-\frac{1-N^2}{2\cos\varphi_{,}} d \sqrt{\frac{2h+1}{2\cos\varphi_{,}}\frac{\lambda_{,}}{R} - 2\frac{\varDelta}{R}} \left\{1 + \frac{\cos^2\varphi + N^2\cos^2\varphi_{,}}{\cos^2\varphi\cos^2\varphi_{,}} tg^2\varphi_{,}\right\}$$

Duo igitur radii vectores in plano incidentiae ducti impares sunt atque eorum differentia est

$$42)\qquad r_{,} - r_{,,} = \frac{1}{2}\, tg\,\varphi_{,}\,(2h+1)\,\lambda_{,}.$$

Quae differentia eo major est, quo major angulus incidentiae, quo major deinde $(2h+1)$, id est haec differentia pro annulis a centro dimotis major est quam pro annulis propinquis; denique haec differentia, ipsa pro quoque valore radii R sphaerae eadem, prae toto radii vectoris valore eo major quare eo magis conspicua est, quo minor R. — Ea igitur mutatio, quam ultimo refractio huic phaenomeni affert, primum pertinet ad communem annulorum dilatationem, deinde membrum correctionale alium valorem accipit. Qua in re posui $\frac{d}{R}$ esse eodem ordine quo $\frac{x_{,}}{R}$. Si tamen d prope $= R$

esset, alii valores quantitatum ξ et η efficerentur, quam qui aequationibus 36) positi sunt; quare alia phaenomena efficerentur.

6. Summa calculi cum observatione comparatur.

Feci observationes microscopio, quod et in velamento quodam levari et circa axem horizontalem circumagi poterat; haec vero axis in baculo verticali movenda erat. Atque primum quidem et in lumine albo et in lumine flavo homogeneo, quod flamma alcoholis cum chlornatrio pariebatur, observationibus factis reperi, ut annuli cernerentur, microscopium in certo quodam situ esse collocandum; quo ex situ si microscopium in velamento paullulum supra vel infra movetur, annuli statim minus clari fiunt moxque omnino evolant e conspectu. Deinde vero contigit mihi, ut id planum constituerem, in quo annuli versarentur; usus enim lamina vitrea, cujus superficies multis rimis tecta erat, apparuit, annulos in medio rimarum versari, annulos igitur aut in superiore aut in inferiore superficie laminae vitreae esse positos. Atque tessera quadam inveni, hanc superficiem superiorem non esse; annuli igitur positi sunt in eo plano, a quo sphaera tangitur. Quod planum cum ob refractionem e situ suo $z = o$ levatum videatur in situm $z = -d$, (Fig. 4), ea assumptio, quam feci pag. 27, plane probata est.

Denique ubi diligentius observavi, apparuit (Fig. 5) radium vectorem versus EC ductum esse majorem quam radium ED [ubi fons luminis in linea EC], annulos igitur esse ellipses excentricas, quarum centra a puncto E versus C dimota essent. Quae excentricitas cum valde parva fuerit, atque ex oculi errore videri potuerit, annulorum radios vectores metitus sum per dua fila decussantia, quae movebantur per cochleam micrometricam. In mensionibus tamen indiligentia quaedam erat, primum quia microscopium non satis immotum stabat, deinde quia ob ambitum annulorum arbitrium quoddam erat, in quonam puncto fila essent collocanda; atque haec indiligentia maxima erat, ubi filum erat collacandum in ipso centro E. Quare non metitus sum (Fig. 5) annulorum radios vectores a puncto E ductos, sed distantiam annuli cujusdam a primo annulo obscuro; qua in re multo major diligentia fieri potuit. Tum novi errores orti sunt, quod nullum criterium erat, fila re vera in plano incidentiae moveri. Quare ex una mensione nihil concludi poterat, sed magnus mensionum numerus erat adhibendus. Atque primum feci mensuras in lenticula biconvexi, in qua distantia foci erat aequalis sex digitibus. Si II_o distantia secundi annuli obscuri a primo annulo in linea *EC*, II_u eadem distantia in linea *ED* etc., inveni, lumine flavo homogeneo adhibito, medium ex duodecem observationibus:

$$II_o \; 62{,}5^\circ \qquad II_u = 62^\circ$$
$$III_o \; 118^\circ \qquad III_u = 111^\circ$$
$$IV_o \; 163^\circ \qquad IV_u = 159^\circ$$
$$V_o \; 209^\circ \qquad V_u = 201{,}5^\circ$$
$$VI_o \; 247^\circ \qquad VI_u = 238^\circ$$

ubi 100° par uni circuitui cochleae micrrometricae; error medius erat in primis annulis = 2°, in magis distantibus = 3°. Atqne apparet, differentiam inter r_o et r_u majorem qum observationis errorem esse atque crescere cum numero annulorum. Denuo in eadem lenticula ad metiendum VI_o viginti observationes feci paremque numerum ad metiendum VI_u, e quibus apparuit

$$VI_o = 245^\circ \qquad VI_u = 235^\circ,$$

medius error observationis = 3°.

Denique alia lenticula usus, in qua distantia foci aequalis tribus digitibus, inveni e quadraginta observationibus

$$VI^o = 168^\circ \qquad VI_u = 160^\circ,$$

medius observationis error = 2°.

Secundum theoriam $VI_o - VI_u$ in utraque lenticula par esse deberet; differentia vero debetur observationis erroribus. Patet autem ex omnibus mensionibus, radium rectorem annulorum versus *EC* ductum majorem esse quam versus *ED* ductum, quamvis differentiam vix metiri possis, id quod plane congruit cum ea theoria, quam supra explicavi. —

Restat, ut inquiram, quonam sint momento iteratae radii luminis in sphaera reflexiones, quas considerandas primus ill. Poisson docuit. Hunc casum cum accurate investigaverim, inveni, ubi ultima refractio non respicitur, formulae (31) membrum primum non mutari, membrum alterum, proportionale λ, cum factore quodam esse multiplicandum, qui prope $= 1$; ad hunc vero exitum, per calculum valde complicatum perveni, quem hoc loco omittam.

Denique quaesivi, quonam modo formulae (31) mutarentur, si non amplius A', B', C' constantes considerantur, sicuti pag. 21 fecimus, sed functiones quantitantum ξ et η; atque inveni, ad formulas (31) novum membrum esse adjiciendum, proportionale λ, sed quod $(2h + 1)\ \pi^2$ in denominatore habeat, ut jam negligi possit prae secundo membro hujus formulae.

Summa dissertationis haec est, ori phaenomenon propositum, si unam in sphaera reflexionem statuimus, duobus radiis interferentibus, quorum alter in lamellam intravit, alter in ipsa superiore lamina reflexus est. Hoc quidem rectum esset, si statuimus, a quoque puncto undae planae incidentis unum radium emitti undae perpendicularem. Re vera autem phaenomenon magis conspicunm est, quam quod duobus radiis interferentibus efficiatur. Quare videtur mihi etiam alia hujus phaenomeni analogia cum phaenomenis diffiactionis esse; neque unum radium a quoque undae incidentis puncto emitti, sed pricipium Huyghenii applicandum existimo, ex quo unda in qualibet situatione ex quaque priore situatione orta judicari potest. —

Tum vero a quoque undae incidentis GG (Fig. 6.) puncto E non unus radius perpendicularis, sed infinitus radiorum obliquorum numerus emittitur, ita ut quodque superioris plani punctum D non uno radio, sed infinito eorum numero secetur; quorum radiorum cum pars interferat, restabunt ii modo radii qui siti sunt in cono quodam et ad quos pertinet undae incidentis zona quaedam zentralis. — Attamen hanc rem in calculo omisi et respexi unum radium perpendicularem, idest pro cono infinite parvo statui coni axem. —

Fig. I.

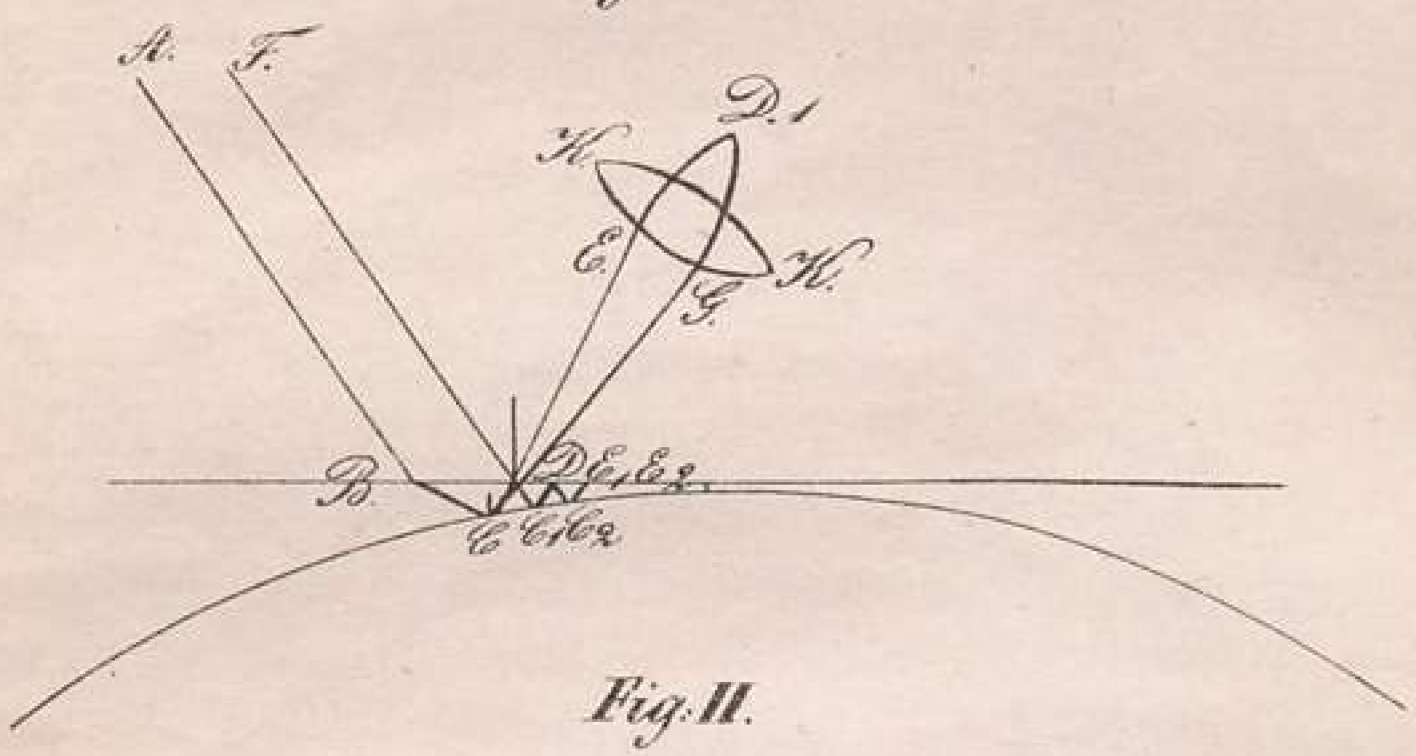


Fig. II.

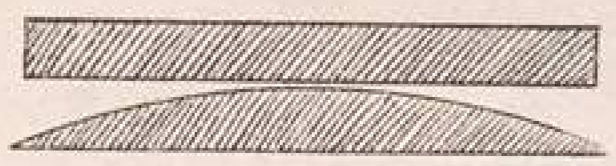


Fig. III.

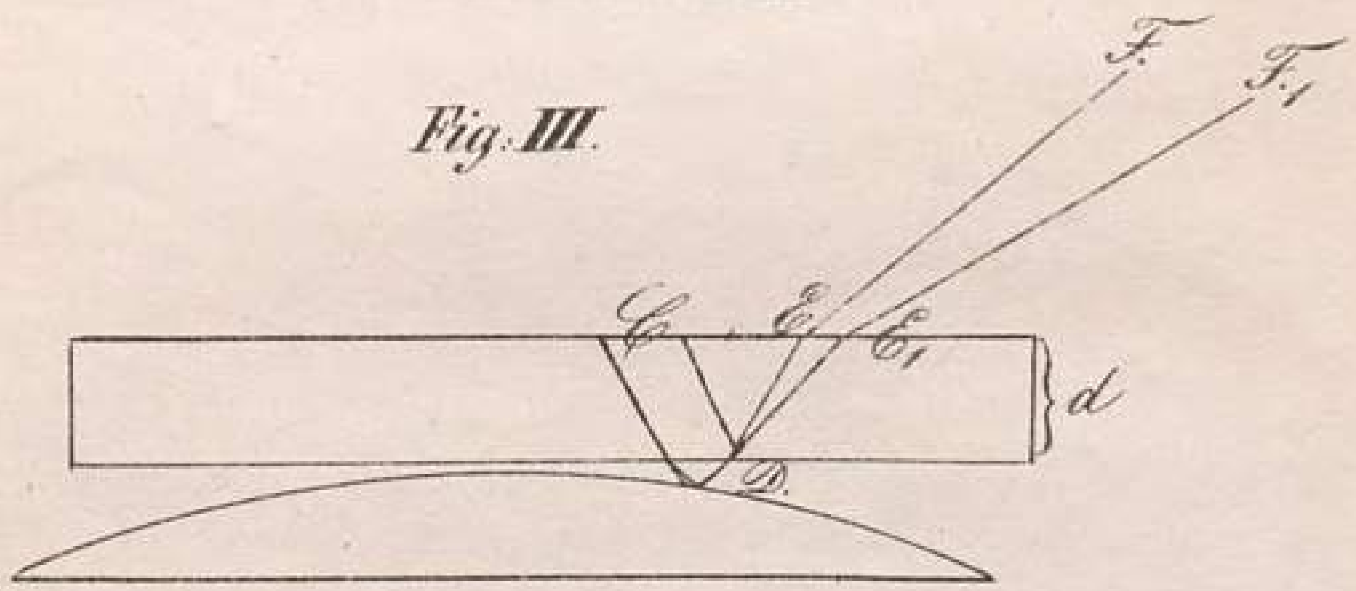


Fig. IV.

Fig. V.

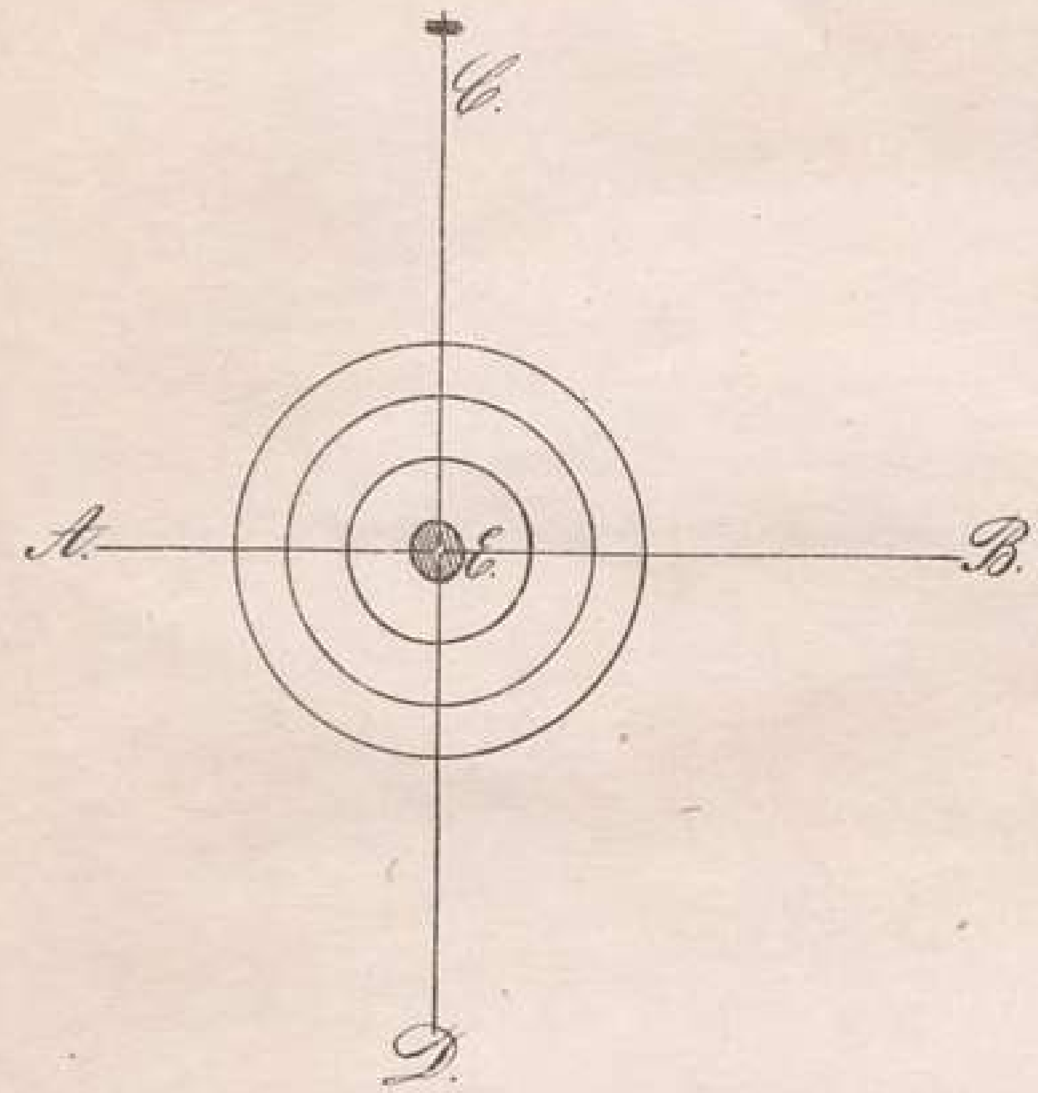
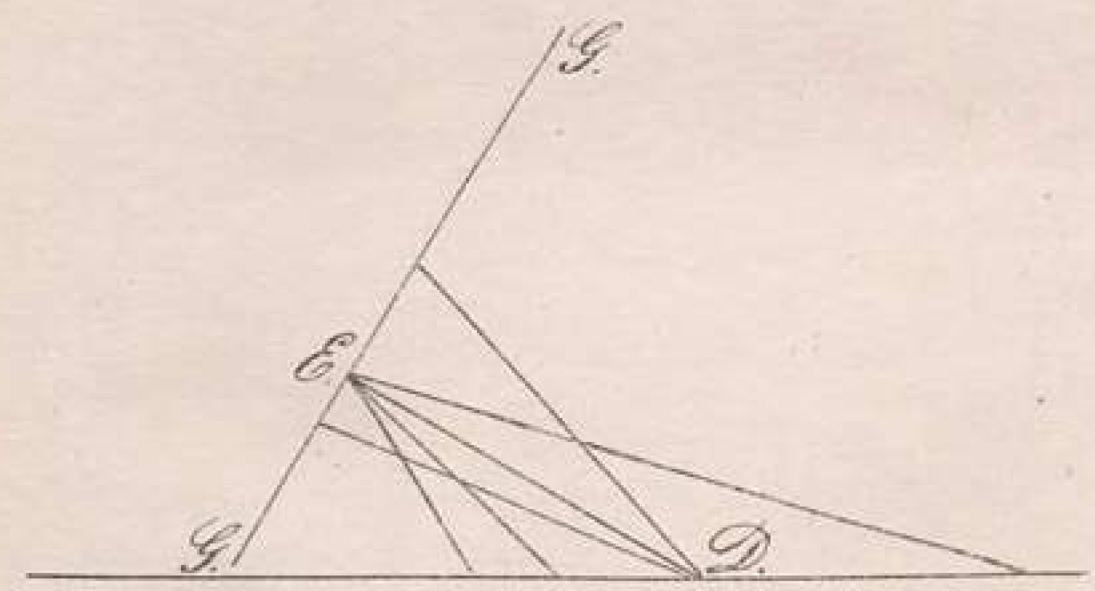


Fig. VI.



VITA.

Natus sum ego Albertus Wangerin Gryphimontii, in urbe Pomeraniae, die XVIII. mensis novembris a. 1844 *patre Henrico matre Emilia e gente Bathke, quos adhuc vivos pie veneror. Fidei addictus sum evangelicae. Primis literarum elementis imbutus, mense Aprili* 1853 *in urbis patriae gymnasium a. cl. directore Campe receptus sum. Ibi per novem annos cum in ceteris disciplinis, quibus aetas puerilis informari solet, a praeceptoribus optimis dilectissimisque institutus sum, tum in mathematica maxime physicaque studia incubui, quibus studiis insigni praeceptoris humanissimi Dietrich opera adjutus, tanto mox matheseos amore sum incensus, ut huic disciplinae totum me dedere constituerem. Itaque vere a.* 1862 *maturitatis testimonio instructus Halas adii, ubi a prof. ill. Jacobi tum rectore magnifico, in civium academiae Fridericianae numerum receptus et a prof. ill. Leo decano in album philosophorum inscriptus sum. Hac autem in alma literarum sede per tria semestria scholis interfui et disserentes audivi hosce: prof. Ill. Heine, C. Neumann, Rosenberger de mathematicis, ill. Knoblauch de physice experimentali, ill. de Schlechtendal de botanice, ill. Erdmann de psychologia.*

Ineunte vero auctumno a. 1863 *Regimontium me contuli, ubi quinque per semestria scholis et seminariis a viris doctissimis F. Neumann et Richelot de rebus physicis el mathematicis habitis interfui. Quibus omnibus viris optime de me meritis gratias nunc ago semperque habebo quam maximas. —*

Theses.

1) Proprium fluidum electricum non est statuendum, sed vires electricae, quae vocantur, statui proprio agitatae materiae seu aetheris tribuendae sunt.

2) In lucis legibus explicandis ratio habenda est earum virium, quas particulae penderabiles in aetherem imponderabilem exercent.

3) Considerationibus geometricis in mathesi plerumque plus assequimur quam calculo.